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A decision theoretic approach towards planning of proof load tests

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ABSTRACT: Accurate determination of the bearing capacity of bridges is of high importance for society. Concerns are raised about the actual bearing capacity of bridges due to aging related deterioration, ever increasing traffic loads and conservative design. Proof load testing is often used for evaluations of bridge capacity. However, extensive proof load tests tend to be costly. Further, the risks of damage to the bridge imply that a proof load test may not always be cost effective. The performance of proof load testing and its outcomes is further dependent on factors such as the chosen loading, the monitoring technology and methods, and the stop criteria. A decision theoretic approach is utilized to demonstrate the optimal strategy for proof load testing procedures and collection of information. The decision scenario constituting the planning and performance of the proof loading is considered along with prevention of damage to the bridge. The decision maker is the proof loading planner who chooses the loading, the monitoring technologies and methods as well as the stop criteria to minimize the expected costs of the test and to comply with acceptable risks. A case study is developed and the optimal strategy with respect to loading, monitoring technology and stop criteria is identified as those with the maximum utility to the decision maker.

1. INTRODUCTION

The accurate determination of bearing capacity of bridges is of high importance to society in view of the considerable life safety and economic consequences involved. Due to the long service life of bridges, there are concerns about the actual bearing capacity due to deterioration processes and increasing traffic loads. On the other hand, it is often observed that the actual bearing capacity of the bridge is higher than the theoretically estimated capacity due to factors not considered in the design such as redundancies and reserve strength or due to conservative design practices. Whatever may be the motivation, accurate determination and classification, of the bridge capacity is linked to benefits as decisions regarding bridge maintenance and load rating can be made based on the information. The classification of bridge capacity based on proof load test information is a subject that has been dealt with in the recent years (Faber et al. 2000b; Lantsoght et al. 2017; Tzyy Shan Lin and Nowak 1984). While bridge reclassification carries considerable monetary benefits for the bridge operator, extensive proof load tests tend to be costly and the risks of damage to the bridge due to the loading suggest that the testing may not always be cost effective. The existing classification framework does not take into account the risks associated with performing the proof testing. Casas and Gomez (2013) identified that the estimation of the loading to apply while being within the acceptable risks as one of the main difficulties in the execution of the proof load test. A step in this direction was taken by Stewart and Val (1999) where the expected costs and risks associated with proof load testing were estimated. In the past years, the development of decision analysis and its application to structural integrity management has lead to the identification of risk informed actions for the management of assets especially in the offshore energy sector (Faber et al. 2000a; Sørensen et al. 1991; Straub and Faber 2005). In this paper, the testing of bridges is analyzed from a decision theoretic perspective with the evaluation of and optimization of the choices available to the proof testing planner with respect to the chosen loading level, the monitoring technology and stop criteria.

2. BRIDGE TESTING AND ASSESSMENT

2.1. Proof testing

The proof testing of bridges is performed to serve two purposes: to confirm the existing load rating of the bridge or to increase the load rating. It is in such cases hypothesized that standard theoretical methods for capacity assessment give conservative results without accounting for the complexities in structural behavior and the exhibition of redundancies and reserve strength. Proof load testing enables the capacity assessment with the identification possible hidden reserves. The use of a proof load test is also recommended in cases where information (drawings) about the bridge, and hence its capacity information is not available.

When performing the proof load test, the choice of the maximum load level is crucial: on the one hand, the higher the chosen load level, the more the information about the bearing capacity that can be obtained but on the other hand, applying a high load leads to the possibility of irreversible damage. To overcome this contradiction, stop criteria are used wherein the structural response is measured against a maximum allowable value to prevent irreversible damage. During the test, the structural response is measured at critical locations through a monitoring system and is analyzed and observed with respect to pre-defined thresholds for damage indication. Guidelines and codes such as for e.g. the German guideline ‘Deutscher Ausschuss für Stahlbeton (DAfStb)’ and the ACI 437.2M-13 provide stop criteria in terms of (for example) requirements for strain in concrete and reinforcement steel, crack widths for new and existing cracks, deflection limits etc., but these stop criteria are provided for buildings and not bridges. Recently, stop criteria based on laboratory and field testing of bridges have been proposed (Lantsoght et al. 2016, 2018).

Additionally, proof testing also requires development of a test rig that can facilitate a precise load application and monitoring systems well adapted to providing accurate measurements in the in-situ environment (Schmidt et al. 2018).

2.2. Reliability based assessment

Probabilistic models and procedures for modelling the performance of a bridge accounting for information from proof load testing have been treated in the last few decades. For example, Faber et al. (2000) provide a reliability based method to estimate the required proof load levels for a bridge reclassification. The Danish Road Directorate provide an extensive guideline for the reliability based assessment of existing bridges (Vejdirektoratet 2004). The procedure involves the modelling of the uncertainties in the loads and resistance of the bridge and the computation of reliability level. A target reliability level for the remaining service life of the bridge is considered such that the risks to human safety and economic costs in case of a failure are minimized. The target reliability level is often provided by the relevant codes. The bridge is then assessed to satisfy the reliability-based criteria. Results from the proof load testing can be directly implemented in updating the (prior) knowledge on bridge assessment by their integration into the probabilistic model updating and refining. The probability based approach for bridge condition assessment and updating have proven to determine a higher capacity than the deterministic approaches, resulting in considerable monetary benefits to the bridge operators by optimizing integrity management actions (Lauridsen et al. 2007).

2.3. Model updating with proof load testing information

The performance of a bridge structure is described probabilistically through the quantification of its probability of failure. The failure probability is obtained with a limit state function of the uncertainties that influence the bridge performance e.g. the resistance, load(s), deterioration etc. The limit state function can be formulated to describe the structural performance at the component level and/or at the system level. At the system level, the limit state function is obtained through a logical system modelling combination depending on the systems model, the number of contributing components and the dependencies between the components. The event of the bridge surviving a certain proof load level can also be modelled with a limit state function and used to update the prior failure probability with Bayesian updating. Considering $P(f_{sys})$ to be the prior failure probability of the bridge and $P\left(f_{PL}(S_{PL_j})\right)$ to be the probability of test failure with load level S_{PL_j}, the updated probability of bridge failure in any year t following a successful proof load test in year t_{PL} is obtained with Bayes' theorem:

$$P\left(f_{sys}^{U}\left(S_{PL_j}, t\right)\right) = \frac{P\left(f_{sys}(t) \cap \overline{f_{PL}}\left(S_{PL_j}, t_{PL}\right)\right)}{P\left(\overline{f_{PL}}\left(S_{PL_j}, t_{PL}\right)\right)} \quad (1)$$

3. DECISION ANALYSIS APPROACH

3.1. Context

As established in the previous section, structural reliability theory is used in conjunction with Bayesian updating to update knowledge on the condition of the bridge utilizing information from a proof load test. However, when it comes to designing and specifying the parameters for performing the proof testing, the decision maker can face a number of choices in e.g. selection of the proof load level, the monitoring system, the stop criteria etc. With the hypotheses that information from different sources can be combined to update the knowledge on the bridge condition, it becomes possible to sketch a decision scenario where the various decision parameters can be optimized with a decision theoretical approach.

3.2. Methodology

The decision of considering yet unknown additional information and additional actions is based upon achieving a positive expected Value of Information and Actions (VoIA) (Brüske and Thöns 2018). The expected VoIA is calculated as the difference between the expected optimal utility with and without additional information and actions. The expected optimal utility is obtained with the maximization of the expected utilities and identification of the optimal action(s) associated with the acquired information as well as the optimal information acquirement strategy. In this paper, the decision scenario regarding performance of a proof load test is considered where the planner of the proof load test chooses the loading, the monitoring system, and the stop criteria. Different loading levels in relation to the characteristic value of the annual maximum live load are chosen. The information acquirement (from the monitoring setup deployed during the proof testing) leads to a measurement of the loading response and hence measurement of the realization of the model uncertainty related to the load effects due to the applied proof load. It is here assumed that the structure behavior during testing is linear and hence a measurement of the loading response i.e. the strains or deformations at some critical locations during the testing can lead to knowledge of the realization of the loading model uncertainty.

The realizations of the model uncertainties are categorized in connection with target failure probabilities for proof loading, following the procedure presented by Agusta & Thöns (2018). If the realization of the model uncertainty is less than the expected value (herein called the threshold), then the implication is that the performance during the testing is as expected. This outcome is linked to the application of a higher loading level. In this way, the target failure probability for proof loading (or the threshold value for the loading model uncertainty) is modelled as the stop criteria. The computation of expected utilities include the expected costs of the testing and monitoring, the expected costs of failure during testing (test risks) and the expected costs over the life cycle of the bridge. With the minimization of expected costs, the optimal strategies for proof loading and monitoring are identified along with the optimal stop criteria. This is demonstrated in the following sections with a case study.

4. CASE STUDY

A generic bridge is considered with the designed service life of 100 years (t_{SL}). It is assumed that the proof loading test is to be performed at 85 years (t_{PL}).

4.1. Decision Scenario

The decision scenario is visualised in Fig. 1 in the form of a decision tree. Here, i_i represents the choice of utilizing a monitoring strategy with Z_k being the outcomes of the monitoring. The actual proof loading is modelled as the action available to the decision maker with the choice of different load levels S_{PL_j} varying from 0.5 to 2 times the characteristic value of the annual maximum live load S_k. The outcomes of the loading are contained in set **Y** and include the events of test failure (Y_1), or success (Y_2). Following the test outcomes, the performance of the bridge is updated, considering the two system states of failure (X_1) and survival (X_2).

A choice between two monitoring systems is considered: an expensive but more precise setup (i_1) against a cheaper setup with a higher measurement uncertainty (i_2). For example, deformations during the testing may be measured with different methods: LVDT's, a digital image correlation (DIC) system etc. A threshold value for the loading model uncertainty related to the load effects due to the applied proof load $\widehat{M}_{S_{PL,th}}$ is defined by linking the component failure probability during proof loading to a target failure probability for the proof loading $P^T(f_{PL})$ (the superscript 'T' implies target probability),

$$P\big(f_{PL}(S_{PL_1})\big|\widehat{M}_{S_{PL,th}}\big) = P^T(f_{PL}) \tag{2}$$

Here, $P\big(f_{PL}(S_{PL_1})\big|\widehat{M}_{S_{PL,th}}\big)$ is the probability of failure of a component due to proof load S_{PL_1} given that the loading model uncertainty is equal to the threshold $\widehat{M}_{S_{PL,th}}$ (which is also to be optimized here).

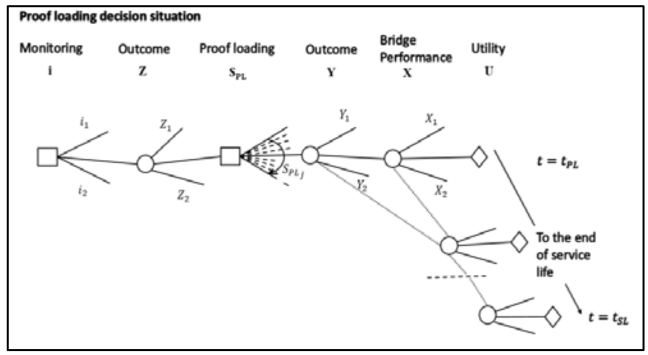

Figure 1 : Illustration of the decision situation

By using the threshold, the outcomes of structural measurement are the two indication events: event Z_1 where the monitoring indicates that the performance in the proof loading situation is as expected and hence the action of a higher loading level can be performed and event Z_2 where the monitoring indicates that the performance is not as expected. Note, that the threshold and the indication events are specifically related to the component model uncertainty representing the uncertainty in the load distribution in the elastic range of the structural system behavior.

The utility distribution **U** models the costs associated with the testing and monitoring and the consequences associated with the test outcomes and bridge life cycle performance. The dimension of time is added from the year of proof loading to the end of the service life of the bridge to illustrate the effect of the proof loading on the expected utilities through the updated (lower) structural risks.

4.2. Probabilistic models

4.2.1. Prior Performance

The failure probability of the bridge is computed by modelling the system as a ductile Daniels' system (Daniels 1945). The system model is considered as a generic model of a redundant structural system and is not based on any actual bridge. The probabilities of failure of a component and the system are:

$$P\left(f_{sys}(t)\right) = P\left(\sum_{i=1}^{n_c} M_{R_i} R_i(t) - M_S(S_D + S_L) \leq 0\right) \quad (3)$$

$$P\big(f_i(t)\big) = P(M_{R_i} R_i(t) - M_S(S_D + S_L) \cdot (\frac{1}{n_c}) \quad (4)$$

$$R_i(t) = R_i(1 - D_i t) \quad (5)$$

In the equations above, R_i is the component resistance, S_L is the annual maximum live load, S_D is the dead load, M_{R_i} and M_{S_L} are the associated model uncertainties, D_i is the component deterioration and n_c is the number of components. The mean of the component resistance is calibrated such that the system reliability without any damages is 4.2 in the Ultimate Limit State with a reference period of 1 year. Here, it is assumed that information about the resistance is known, but this is not typically the case in the proof testing situation, as has been discussed in section 2.1. The deterioration D_i is assumed to follow a Lognormal distribution with a mean of 0.001 and a standard deviation of 0.001 (Thöns et al. 2018). The system is modelled with 5 components, considering the following correlation models between the components' parameters : $\rho_{R_i R_j} = 0.7, \rho_{M_{R_i} M_{R_j}} = 0.5, \rho_{D_i D_j} = 0.8$.

Table 1: Probabilistic Model for structural properties

Parameter	*Distribution*	*Mean*	*CoV*
R	Lognormal	Cal.	10%
S_D	Normal	1.0	5%
S_L	Gumbel	1.0	10%
S_{PL}	Deterministic	0.5-2.0S_k	-
M_R	Lognormal	1.2	12.5%
M_S	Lognormal	1.0	20%
$M_{S_{PL}}$	Lognormal	1.0	20%

The probabilistic model presented here is generic and developed to demonstrate the proposed approach.

4.2.2. Modelling of test outcomes

The outcome of failure of the bridge due to the proof loading with the j^{th} load S_{PL_j} is modelled with the limit state function,

$$P\left(f_{PL}\left(S_{PL_j}, t_{PL}\right)\right) = \\ = P\left(\sum_{i=1}^{n_c} M_{R_i} R_i(t_{PL}) - M_{S_{PL}}\left(S_D + S_{PL_j}\right) \leq 0\right) \quad (6)$$

In the year of performing the proof load test, the bridge may fail due to the annual maximum live load or due to the proof loading test. The annual failure probability $P_{f_{ann}}$ in that year, modelled as the union of the events of failure due to test or due to annual maximum live load (conditional on test survival), is:

$$P\left(f_{ann}\left(S_{PL_j}, t_{PL}\right)\right) \\ = P\left(f_{sys}(t_{PL}) \middle| \overline{f_{PL}}\left(S_{PL_j}, t_{PL}\right) \cup f_{PL}\left(S_{PL_j}, t_{PL}\right)\right) \quad (7)$$

In any year t following a successful test, the probability of failure of the bridge is updated according to Eq. (1). It should be noted here that the knowledge that the bridge has survived upto the present year is not used in updating its reliability.

4.2.3. Modelling of monitoring information

Considering a threshold value for the model uncertainty $\widehat{M}_{S_{PL},th}$ corresponding to a certain target failure probability for proof loading (Eq. 2), the probability of the indication Z_1, is modelled with,

$$P(Z_1) = \int_0^{\widehat{M}_{S_{PL},th}} f_{M_{S_{PL}}}(m_{S_{PL}}) \mathrm{d}m_{S_{PL}} \quad (8)$$

Following the indication Z_1, the action of a higher loading level is performed. If the monitoring indicates inadequate performance (Z_2), then risks of performing the test is too high and the proof loading should not be performed. Following the indication Z_1, the probability of failure of the proof load test is calculated,

$$P\left(f_{PL}\left(S_{PL_j}, t_{PL}\right)\middle|Z_1\right) = P\Big(\sum_{i=1}^{n_c} M_{R_i} R_i(t_{PL}) - UM_{S_{PL}}|_0^{\widehat{M}_{S_{PL},th}}\left(S_D + S_{PL_j}\right) \le 0\Big) \qquad (9)$$

Here, U is the measurement uncertainty of the monitoring system, modelled as a normal distributed random variable with mean of 1.0 and standard deviation 0.01 (i_1) and 0.03 (i_2), respectively, and $M_{S_{PL}}|_0^{\widehat{M}_{S_{PL},th}}$ represents the random variable model uncertainty between the interval 0 and $\widehat{M}_{S_{PL},th}$.

4.3. Utility quantification

The expected utilities are quantified with the accumulation of the risks over the remaining service life of the bridge. The (annual) risks of structural failure are obtained with the multiplication of the annual probability of failure with the failure consequence, which is considered as being equal to the cost of construction of the bridge (C_f). In computing the utilities with the testing and monitoring, the cost of testing and monitoring is also added. The costs associated with the proof loading (C_{PL}) is modelled as a 0.1% of bridge cost and the monitoring systems are modelled to cost 0.01% (i_1) and 0.005%(i_2)of the bridge cost, respectively. A depreciation in the modelled costs is considered to discount the future costs to present value, with a discount rate of *r* =2%. The utility calculation is elaborated upon with Eq. (10-12) where the calculation without any proof loading (Eq. 10), with loading but without monitoring information (Eq. 11) and with both monitoring and loading (Eq. 12) is presented.

$$U_0 = \sum_{t=t_{PL}}^{t=t_{SL}} P\left(f_{sys}(t)\right) \cdot C_f(t) \qquad (10)$$

$$\begin{aligned} U\left(S_{PL_j}\right) = {} & C_{PL} + P\left(f_{ann}\left(S_{PL_j}, t_{PL}\right)\right) \cdot C_f(t_{PL}) \\ & + P\left(\overline{f_{PL}}\left(S_{PL_j}, t_{PL}\right)\right) \\ & \cdot \sum_{t=t_{PL}}^{t=t_{SL}} P\left(f_{sys}^{U}\left(S_{PL_j}, t\right)\right) \cdot C_f(t) \end{aligned} \qquad (11)$$

$$\begin{aligned} U\left(i_i, S_{PL_j}^{*}\right) = {} & C_{PL} + C_{i_i} + P(Z_1) \\ & + \min_{S_{PL_j}} \Bigg(P\left(f_{ann}\left(S_{PL_j}, t_{PL}\right)\middle|Z_1\right) \cdot C_f(t) \\ & + P(\overline{f_{PL}}(S_{PL_j}, t_{PL})|Z_1) \\ & \cdot \sum_{t=t_{PL}}^{t=t_{SL}} P(f_{sys}^{U}(S_{PL_j}, t)|Z_1) \cdot C_f(t) \Bigg) \\ & + P(Z_2) \\ & \cdot \sum_{t=t_{PL}}^{t=t_{SL}} P\left(f_{sys}(t)\right) \cdot C_f(t) \end{aligned} \qquad (12)$$

4.4. Results

The expected utilities '$U\,(S_{PL_j})$' are calculated and presented (in blue) as a function of the loading level in Figure 2. The difference between the expected utilities with and without proof loading 'U_0 - $U\,(S_{PL_j})$' is plotted with green in the figure.

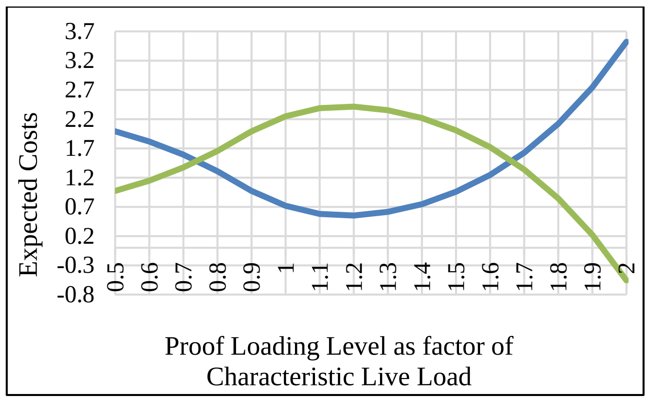

Figure 2: Plot of the expected costs (blue) and expected benefit gain i.e. difference between the expected costs with and without loading (green) as a function of the proof loading level

From the figure, it is observed that the expected costs show a reduction with the increasing loading level up to a point beyond which the risks from the testing exceed the potential life-cycle risk reduction. The optimal loading level is 1.2 times the characteristic value of the annual maximum live load. The annual failure probability with proof loading with this load level is ~1.00E-07 (in the year of proof loading). Without any proof loading, the annual (prior) probability of failure in this year is 1.29E-03.

The threshold values for the loading model uncertainty are computed for different target failure probabilities for proof loading (refer to Eq. 2). These are presented in Table 2 along with the probability of the indication event Z_1.

Table 2 : Threshold values and indication probabilities for monitoring systems

	Monitoring System i_1		Monitoring System i_2	
$P^T(f_{PL})$	$\widehat{M}_{S_{PL},th}$	P_{Z_1}	$\widehat{M}_{S_{PL},th}$	P_{Z_1}
1.00E-02	1.83	0.999	1.83	0.999
8.50E-03	1.78	0.998	1.77	0.998
8.00E-03	1.75	0.998	1.75	0.998
7.50E-03	1.73	0.997	1.73	0.997
7.00E-03	1.71	0.997	1.70	0.997
6.50E-03	1.68	0.996	1.68	0.996
6.00E-03	1.65	0.995	1.65	0.995
5.50E-03	1.62	0.994	1.62	0.994
5.00E-03	1.58	0.992	1.58	0.991
4.50E-03	1.54	0.988	1.54	0.988
4.00E-03	1.49	0.982	1.49	0.982
3.50E-03	1.43	0.971	1.43	0.970
3.00E-03	1.36	0.949	1.36	0.948

Next, the expected utilities $U(i_i,S^*_{PL_j})$ for a monitoring strategy and corresponding to the respective threshold values are calculated along with the identification of optimal load level $S^*_{PL_j}$. Finally, the expected value of information and actions is calculated for the two monitoring strategies as '$U_0 - U(i_i,S^*_{PL_j})$' and presented in Figure 3 as a function of the different stop criteria. The threshold value with $P^T(f_{PL})$: 5.00E-03 (i_1) and $P^T(f_{PL})$: 4.50E-03 (i_2) is identified as leading to the highest expected value gained for the respective strategies. Between the two monitoring strategies, i_1 demonstrates higher expected benefits. The expected costs with the indication of Z_1 and corresponding to the respective optimal threshold levels is plotted in Figure 4 along with the expected costs with only loading.

5. DISCUSSION AND CONCLUSION

A decision analytical approach for the planning and performance of a proof load testing is developed and demonstrated. A pre-posterior

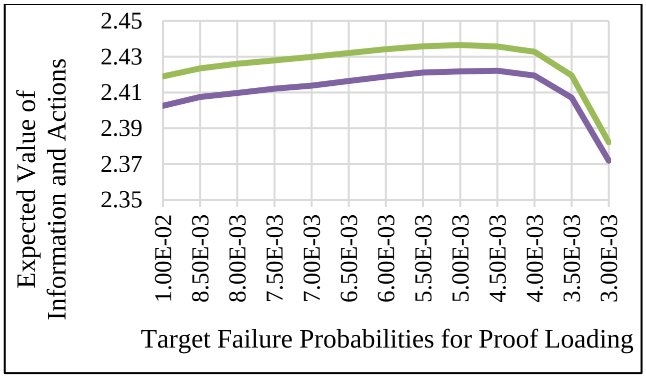

Figure 3: Illustration of the Expected Value of Information and Actions from the two monitoring strategies i_1 (green) and i_2 (purple) as a function of the target failure probabilities.

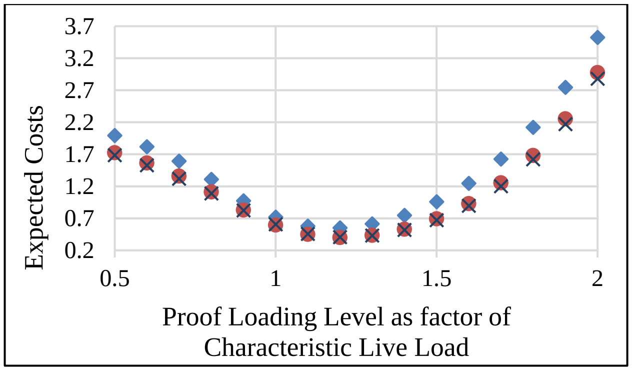

Figure 4: Illustration of the Expected Costs given Z_1 from the two monitoring strategies i_1 (orange circle) and i_2 (blue cross) and the expected costs with only loading '$U(S_{PL_j})$' (blue diamond) as a function of the loading level.

decision analysis is considered for the identification of the optimal loading level, monitoring strategy and stop criteria. A case study with respect to a generic bridge is presented and the optimal decision parameters are identified. It is demonstrated how the monitoring and testing information can be used with a stop criteria to identify decision rules for a safe and efficient proof load testing.

The identified optimal proof loading level is low: this may be because we have assumed a known stochastic model for the resistance. It is expected that a higher optimal loading level may be identified if the analysis is performed for the case where the resistance is not known.

The approach and results can be supplemented with the further development of models reflecting bridge performance and with the quantification of measurement uncertainties in relation to an actual monitoring setup for the testing along with an estimation of the related costs.

The decision theoretical approaches will be further developed to align with specific decision scenarios and additional structural information sources such as e.g. sophisticated structural analyses models. In addition, to be considered in the approach is to perform the optimization such that the code constraints on maintaining a target reliability level for the remaining service life are satisfied. Further, the models will be developed to consider also the bias in the resistance model uncertainty to align with the situation where the bridge resistance is expected to be larger than the design value.

6. ACKNOWLEDGEMENTS

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